The Physics of Anvils

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At first glance, the primary system that the blacksmith uses to shape metal, the hammer and the anvil, seem to be simple and not demanding of deep analysis. The anvil is an immovable, non-deformable object, and the hammer squashes the target with the force of its blow. If it is necessary to do more work or larger jobs, just use a bigger hammer and/or swing it faster. But, should the anvil be correspondingly larger? That seems to make sense, since the anvil could no longer be considered immovable if the hammer were large enough. In fact, the anvil might even be damaged. How big is efficient (or efficient enough), and how big is safe (or safe enough)? Do other factors (anvil shape or mounting, for example) influence the process in important ways? It is intuitive that they do, and application of physical insight to this problem can tell not only the qualitative answers to these questions, but also provide quantitative estimates, so we can calculate, for example, what hammer anvil mass ratio is necessary to obtain at least a 95% forging efficiency.

The behavior of objects when they collide is governed by the principles of conservation of momentum and energy. The latter applies for elastic collisions, for which kinetic energy is fully returned to the masses after collision. Conservation of momentum applies in both elastic and inelastic (where some of the collision energy is converted to heat or deformation) collisions. This principle may be codified in the following equation:

$$m_1V_1 + m_2V_2 = m_1v_1 + m_2v_2 \qquad (1)$$

The masses involved in the collision are m_1 and m_2, and the initial and final velocities are V_1, V_2, v_1, and v_2, respectively. The subscripts 1 and 2 refer to the two bodies which for our purposes may be considered to be the hammer and the anvil. Note that there is only one equation in the two unknowns, v_1, and v_2. Therefore, another condition is required to fully determine the system. The degree of freedom will be, following Newton, embodied in the coefficient of restitution as e, the ratio between the relative speed after and before impact. In other words:

$$e(V_1 - V_2) = V_2 - V_1 \qquad (2)$$

For elastic collisions in which no energy is lost, this corresponds to the familiar situation of a moving marble striking a stationary one of equal mass. Here, $e = 1$, $m_1 = m_2$, and $V_2 = 0$, which Equations (1) and (2) lead to $v_2 = V_1$ and $v_1 = 0$. In other words, after the collision m_2 moves off at m_1's initial velocity, and m_1 stops dead. Since the anvil is free to mover after being struck by the hammer, the system corresponds to a freely suspended anvil, rather than one affixed to a stump. Due to this idealization, the results of the calculation would be expected to be pessimistic. In other words, they would provide an upper bound on energy loss, or conversely, a lower bound on efficiency. This is still very useful to us, since it gives a worst case loss for a poorly attached anvil. Calculation of a value which accounts for the attachment to a secondary object, such as a stump, will require more powerful mathematical techniques, and is left to a future article.

From these equations, we can derive two different measures of hammer-anvil system efficiency. The first of these is the oft-quoted rebound height. The use of this measure corresponds to an elastic collision (assuming no loss of energy) in which the final rebound height as a fraction of the initial hammer height is the quantity of interest. In order to write an expression for this quantity, the equation of conservation of energy applies, since the collision is assumed to be lossless.

$$(1/2)\ m_1V_1^2 = (1/2)m_1v_1^2 + (1/2)m_2v_2^2 \qquad (3)$$

where $v_2 = 0$ because the anvil is initially at rest. Combining this with Equation (1) gives:

$$v_1 = -V_1\ [\ 1 - 2m_1/(m_1 + m_2)] \qquad (4)$$

Or, in terms of energy,

$$(1/2)m_1v_1^2 = (1/2)m_1V_1^2\{1\text{-}2m_1/(m_1 + m_2)] \qquad (5)$$

and the efficiency, which is the ratio of final to initial energy, or height, since height and gravitational potential energy are proportional may be written

$$\in_1 = [1 - 2m_1/(m_1 + m_2)]^2 \qquad (6)$$

This quantity, $\in_1$, represents the fraction of the rebound height to the initial height of a dropped hammer. For very small values of hammer to anvil mass ratios, this may be approximated according to a truncated Taylor series as

$$\in_1 = 1 - 4\alpha \qquad (7)$$

where α is m_1/m_2, the hammer-anvil mass ratio. The concept of Taylor series approximation is covered in any elementary calculus text where more details may be found, if needed. In this case, if the hammer weighs 2 lb and the anvil weighs 100 lb, $\in_1 = 92\%$. Note that as long as the anvil mass is not infinite, its subsequent motion, albeit tiny, will steal a bit of rebound height and the efficiency will always be less than unity. In other words, the hammer will always rebound to a slightly lower height than it was dropped from. This efficiency relates to the familiar ball bearing test in which the decrease in rebound height from dropped height is measured. In practice, this test is not solely affected by the hammer-anvil mass ratio, since the assumption of conservation of energy is not quite accurate. Softer steel anvil faces deform slightly upon impact, absorbing more energy than calculated from Equation 6. Thus, the hardness of the anvil surface also affects the rebound height. Rather than complicating the interpretation of energy loss, this test provides a simultaneous, although convoluted, measurement of both the effect of anvil mass and surface hardness on the energy returned to the hammer.

An alternative measure of anvil-hammer efficiency corresponds to the case in which $e = 0$, that is for a purely inelastic collision. In this case, the hammer sticks to the target, and $v_1, = v_2$, so Equation (1) may be rewritten as

$$m_1V_1 = (m_1 + m_2)v_1 \qquad (8)$$

Efficiency is now the energy deposited (lost) into the target. All movement after the collision (movement of the hammer+target+anvil together) represents energy which is not used for forging. The fractional efficiency is now written as

$$\in_0 = (E_i - E_f) / E_i \tag{9}$$

where E_i and E_f are the initial and final energy respectively.

$$E_f = (1/2)\ (m_1 + m_2)v_1^2 = (1/2)\ (m_1 + m_2)\ [m_1V_1/(m_1 + m_2)]^2 \tag{10}$$

after eliminating v1 by using Equation (9). Substituting Equation (10) into Equation (9) gives

$$\in_0 = 1 - m_1/(m_1 + m_2) \sim 1 - \alpha \tag{11}$$

This is a much more optimistic formula than the one for $\in_1$. In the earlier example of a 2 lb hammer and a 100 lb anvil, $\in_0$ is 98%, versus 92% for $\in_1$.

So, which is the more appropriate formula? Blacksmithing processes are somewhat in between full rebounding and full sticking on impact. Most industrial hot forging practices take place at values of e which are quite small, about 0.1 to 0.2 (see Reference 1), and it would seem that $\in_0$ is more indicative of forging efficiency. In a blacksmith's practice, the goal is to move metal, not to make noise and have the hammer rebound high. Therefore, if the purpose of the process is to deform hot metal, the low restitution coefficient for $\in_0$ is the more appropriate one.

This conclusion is actually further reaching than one might expect from simply comparing the formulas and the relative magnitudes of their results. Closer examination of the applicability of each shows that $\in_1$, corresponding to the rebound test, is more heavily influenced by the hardness of the anvil. As will be seen in the following section, hardened anvil tops have yield stresses similar to the maximum stress imposed by ball bearing or direct hammer strikes. Thus, this test is more indicative of the hardness of the hammer and anvil faces than it is is for their suitability for hot forging. This can be readily illustrated by considering the application of the ball bearing test to a sledge hammer face and a large block of mild steel or a vertically oriented section of railroad rail. The sledge hammer face will always score higher on the rebound test, but makes an inferior anvil compared to the above two alternatives. Provided that the anvil is not getting damaged, the dissipation based $\in_0$ provides a more direct indication of energy deposited in the target than does the rebound based . As Brian Brazeal so succinctly stated when demonstrating his very effective Easy Smith mild steel block anvil, “Rebound should matter, but it doesn't.” (Reference 2)